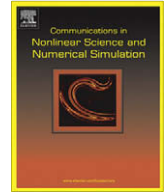




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## Switched control of a nonholonomic mobile robot

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## ABSTRACT

We present a switched control algorithm to stabilize a car-like mobile robot which possesses velocity level nonholonomic constraint. The control approach rests on splitting the system into several second-order subsystems and then stabilizing the system sequentially using finite-time controllers, finally resulting in the mobile robot being moved from one point to another point. State dependent switching control is employed in which the controllers switches on a thin surface in the state-space. Robustness analysis is presented by redefining the switching signal using *relaxed switching surface*. Both, non-robust and robust controllers are validated through numerical simulation.

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## 1. Introduction

Control of nonholonomic systems present greater challenges than systems without constraints. A comprehensive literature survey on nonholonomic systems can be found in [1]. A particular class of these systems are not linearly controllable around any of its equilibrium points and further it cannot be stabilized by smooth feedback control law [2]. Typical examples are mobile robot, surface vessel and space robots. However, the existence of nonlinear controllers is guaranteed from non-linear controllability results. Several control methods have been proposed to stabilize these system using discontinuous controllers based on sliding mode [3,4], hybrid technique [5,6],  $\sigma$  process [7,8], and time-varying controllers [9]. Discontinuous controllers results in the convergence of the closed-loop trajectories while time-varying controllers can guarantee exponential stability but with low rate of convergence and oscillating trajectories.

Switched control system has recently received much attention due to its applicability to a wide range of control problems [10]. Many physical systems exhibits switching in nature. Switched systems can be classified into state-dependent and time-dependent systems. This approach has been applied to nonholonomic systems in [11,6,12]. The stability analysis of the switched systems using Lyapunov functions has been extensively studied.

In this paper, we consider a point-to-point control problem of a car-like mobile robot with a velocity level nonholonomic constraint. Discontinuous controllers are proposed for this problem in the literature using various techniques [13,7] and time-varying controllers are proposed in [9,14]. We propose a state-dependent switched controller to move the mobile robot from one point to an another in the configuration space. We first split the whole system into several second-order systems to perform a specific maneuver and then each of these systems are stabilized sequentially by second-order finite-time controllers. The switching signal is generated by a state-dependent decision maker, thus a state-dependent switching control. In

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each stage the system reaches a subspace of the whole space, finally moving to the origin. We prove the convergence of the closed-loop system to the equilibrium point.

The rest of the paper is organised as follows: Section 2 introduces some mathematical preliminaries along with the model of the mobile robot. Controller design is presented in Section 3 while the stability and robustness are discussed in Section 4. Simulation results and conclusion are presented in the subsequent sections.

## 2. Mathematical preliminaries and background

In this section, we briefly discuss the notion of switched systems, finite-time controllers and the dynamic model of the mobile robot. Consider a nonlinear system of the form

$$\begin{aligned}\dot{x} &= f_p(x), \quad x \in \mathbb{R}^n, \\ x(0) &= x_0,\end{aligned}\tag{1}$$

where,  $p \in \mathcal{P} \triangleq \{1, 2, 3, \dots, l\}$ ,  $\mathcal{P}$  being an index set and is finite. Further, let  $\phi_t^p$  denote the flow associated with the vector field  $f_p$  with the property that  $x(t) = \phi_t^p$  satisfies (1). For the state-dependent switching, the switching signal is a piecewise continuous map  $\sigma: \mathbb{R}^n \rightarrow \mathcal{P}$ . Several subsets are defined in the state-space named as switching surface to define each of the  $f_p$ 's. We next briefly present an overview of finite-time controllers.

Finite-time controller are nonlinear controllers which can drive any arbitrary state to the desired state (assumed as the zero state) in finite-time and stabilize around it. Several finite-time controllers are proposed in the literature in which a first order finite-time controller is explained here. Consider a first-order control system of the form  $\dot{x} = u$ , then the following control laws

$$\begin{aligned}u^1 &= -k_1 x^{1/3}, \quad k_1 > 0 \\ u^2 &= -k_2 \text{sign}(x), \quad k_2 > 0\end{aligned}$$

can drive the system trajectory to zero in finite-time [16,18]. Controller  $u^1$  is a fractional power controller and its stability analysis can be found in [16]. Controller  $u^2$  is a special form of the sliding mode controller with  $x = 0$  as the sliding surface.

Since we split the mobile robot system as composed of several double integrators with each executing specific tasks in this paper, we concentrate on the finite-time controllers which stabilize double integrator. The following double integrator

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= u\end{aligned}$$

is finite-time stable with the following four controllers.

$$\begin{aligned}u^1 &= -\text{sign}(x_1)|x_1|^a - \text{sign}(x_2)|x_2|^b \\ u^2 &= -\text{sign}(x_2)|x_2|^{1/3} - \text{sign}\left[\left(x_1 + \frac{3}{5}x_2^{5/3}\right)\right]\left|x_1 + \frac{3}{5}x_2^{5/3}\right|^{1/5} \\ u^3 &= -\gamma_1 \text{sign}(x_2 + \gamma_2|x_1|^{1/2} \text{sign}(x_1)) \\ u^4 &= -\text{sign}(x_2)|x_2|^{1/3} - \text{sign}\left[\left(\sin(x_1 + \frac{3}{5}x_2^{5/3})\right)\right]\left|\sin(x_1 + \frac{3}{5}x_2^{5/3})\right|^{1/5}\end{aligned}$$

where,  $b \in (0, 1)$ ,  $a > \frac{b}{2-b}$ ,  $\gamma_1, \gamma_2 > 0$ . The control law  $u^1$  is proposed in [16] with finite-time convergence analysis using a Lyapunov function. Controller  $u^2$  is presented in [19], while  $u^3$  is called a second-order sliding mode controller as the states  $x_1 = 0$  and  $x_2 = 0$  can be viewed as sliding surfaces [17]. Finally,  $u^4$  is a special form of  $u^2$  which is also proposed in [19] to especially stabilize the rotational double integrator to avoid the winding phenomenon. In brief, if  $x_1 \in S^1$ , the controller  $u^4$  is useful and finds use in this paper. Controllers  $u^2, u^3, u^4$  have been proved that they are robust with respect to small parameter uncertainties. For details, the reader can refer to [15,17].

Next consider a car-like mobile robot as shown in Fig. 1. The kinematic model of the mobile robot can be written as:

$$\begin{aligned}\dot{x} &= v \cos \theta, \\ \dot{y} &= v \sin \theta, \\ \dot{\theta} &= \omega,\end{aligned}$$

where, the triple  $(x, y, \theta)$  denotes the position and the orientation of the vehicle with respect to the inertial frame and  $v, \omega$  are the linear and angular velocities of the mobile robot. The velocity level nonholonomic constraint imposed on the robot is given by  $\dot{x} \sin \theta - \dot{y} \cos \theta = 0$ . The dynamic model is obtained using the following relations

$$\begin{aligned}M\dot{v} &= F, \\ I\dot{\omega} &= \tau,\end{aligned}\tag{2}$$

where,  $M$  is the mass of the vehicle,  $I$  is moment-of-inertia,  $\tau = \frac{l}{r}(\tau_1 - \tau_2)$ ,  $F = \frac{l}{r}(\tau_1 + \tau_2)$ , with  $L$  being the distance between the center-of-mass and the wheel,  $\tau_1, \tau_2$  are the left and right wheel motor torques and  $r$  is the radius of the rear wheel. The

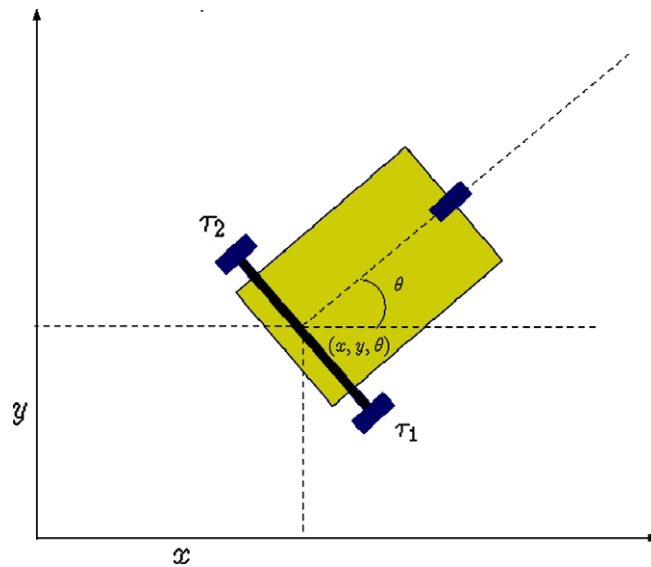


Fig. 1. Schematic of the mobile robot.

equations of motion using the state transformation  $\mathbf{x} = (x_1, x_2, x_3, x_4, x_5) \triangleq (\theta, x \cos \theta + y \sin \theta, x \sin \theta - y \cos \theta, \omega, v)$  can be expressed as a standard form of nonlinear affine control system as follows

$$\dot{\mathbf{x}} = f(\mathbf{x}) + g_1(\mathbf{x})\tau + g_2(\mathbf{x})F$$

$$f(\mathbf{x}) = \begin{pmatrix} x_4 \\ x_5 - x_4x_3 \\ x_4x_2 \\ 0 \\ 0 \end{pmatrix}; \quad g_1(\mathbf{x}) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ k_1 \\ 0 \end{pmatrix}; \quad g_2(\mathbf{x}) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ k_2 \end{pmatrix} \tag{3}$$

where  $f, g_1, g_2$  are smooth vector fields defined on the smooth manifold  $\mathcal{M}_1 = \mathbf{S}^1 \times \mathbb{R}^4$ , with  $k_1 \triangleq \frac{1}{l} > 0, k_2 \triangleq \frac{1}{M} > 0$ .

### 3. Controller design

The control objective is to move the mobile robot from any initial position with initial velocity  $(x(0), y(0), \theta(0), v(0), \omega(0))$  to desired position with zero velocity  $(x_d, y_d, \theta_d, 0, 0)$ . This, without loss of generality, can be considered as the stabilization of (3) to zero. From the physical point of view, moving the vehicle from one point to another point can be done as follows.

1. Stop the robot.
2. Home towards the target.
3. Move to the target.
4. Rotate to the desired orientation.

This control strategy is realized by splitting the whole system into several second order system to execute each tasks. The individual tasks can be considered as the following control problem.

#### 3.1. Stage 1: Stopping the robot

Without loss of generality, we assume that the robot starts with non-zero initial velocity. Then, the first objective is to stop the vehicle, that is, to move the system from  $\mathcal{D}_1 \triangleq \mathcal{M}_1$  to  $\mathcal{D}_2 \triangleq \{x \in \mathcal{M} : x_4 = x_5 = 0\}$ . To achieve this, consider the equations related to  $x_4, x_5$  dynamics

$$\begin{aligned} \dot{x}_4 &= k_1 \tau \\ \dot{x}_5 &= k_2 F \end{aligned}$$

and the following control law

$$\begin{aligned} \tau &= -K_1 \text{sign}(x_4) |x_4|^{1/3}, \\ F &= -K_2 \text{sign}(x_5) |x_4|^{1/3}, \end{aligned}$$

where,  $K_1, K_2 > 0$  are free. The closed-loop system at this stage becomes

$$\begin{aligned}\dot{x}_1 &= x_4, \\ \dot{x}_2 &= x_5 - x_4 x_3, \\ \dot{x}_3 &= x_4 x_2, \\ \dot{x}_4 &= -k_1 K_1 \text{sign}(x_4) |x_4|^{1/3}, \\ \dot{x}_5 &= -k_2 K_2 \text{sign}(x_5) |x_5|^{1/3}.\end{aligned}\tag{4}$$

Then there exist a finite time  $T_1 \geq 0$  such that  $x_4 = x_5 = 0$ , and hence the closed-loop trajectories reach the manifold  $\mathcal{D}_2 \triangleq \{x : x_4 = x_5 = 0\}$  for  $t \geq T_1$ . At the end of this stage the robot is brought to a halt in finite-time.

### 3.2. Stage 2: home towards the target

In this stage, the control objective is to rotate the robot using the torque  $\tau$  to home towards the target point  $(x_d, y_d)$ . From (3), this can be considered as the regulation of the state  $x_3$  to zero. To achieve this objective, we consider the following transformation

$$x_3 = x \sin \theta - y \cos \theta$$

from which it is easy to compute the desired  $\theta_t$  to render the state  $x_3 = 0$ . In other words

$$0 = x \sin \theta_t - y \cos \theta_t$$

can be achieved if  $\theta_t = \tan^{-1} \left( \frac{y}{x} \right)$ . Thus, in this stage, the regulation to the target direction  $\theta_t$  is achieved using the following state equation

$$\begin{aligned}\theta_t - x_1 &= x_4, \\ x_4 &= k_1 \tau.\end{aligned}$$

Applying the same control action as designed in the first stage to prevent the forward motion of the vehicle, hence  $\theta_t$  is a constant. For the torque  $\tau$ , consider the following control law

$$\tau = -\text{sign}(x_4) |x_4|^{1/3} + \text{sign} \left[ \left( \sin(x_{1t} + \frac{3}{5} x_4^{5/3}) \right) \right] \left| \left( \sin(x_{1t} + \frac{3}{5} x_4^{5/3}) \right) \right|^{1/5},$$

where,  $x_{1t} = \theta_t - x_1$ . The above control action regulates the state  $x_1$  to  $x_{1t}$  in some finite-time, hence  $x_3 = 0$  in some finite time  $T_2 > 0$ . In this stage the closed-loop system becomes

$$\begin{aligned}\dot{x}_1 &= x_4, \\ \dot{x}_2 &= x_5 - x_4 x_3, \\ \dot{x}_3 &= x_4 x_2, \\ \dot{x}_4 &= k_1 \left[ -\text{sign}(x_4) |x_4|^{1/3} + \text{sign} \left[ \left( \sin(x_{1t} + \frac{3}{5} x_4^{5/3}) \right) \right] \left| \left( \sin(x_{1t} + \frac{3}{5} x_4^{5/3}) \right) \right|^{1/5} \right], \\ \dot{x}_5 &= -k_2 K_2 \text{sign}(x_5) |x_5|^{1/3}.\end{aligned}\tag{5}$$

Thus, for  $t \geq T_1 + T_2$ , the trajectories reach the set  $\mathcal{D}_3 = \{x : x_3 = x_4 = x_5 = 0\}$ .

### 3.3. Stage 3: move to the target

In this state the robot is performing pure forward motion to reach the target point. This can be considered as regulating the state  $x_2$  to zero in (3). Consider the dynamics of  $x_2$

$$\begin{aligned}\dot{x}_2 &= x_5 \\ \dot{x}_5 &= k_2 F\end{aligned}$$

and note that this dynamics is valid only if  $x_3 = x_4 = 0$  for the entire maneuver. Also, note that at the end of each state the robot is brought to a halt, that is,  $x_3 = x_4 = 0$ . It is necessary to regulate one of the velocities to zero for all stages beyond the first one. Therefore, in this stage, the torque control is designed to maintain the velocity  $x_4 = 0$  which results in  $x_3 = 0$  and further helps to steer the robot in the forward direction. The force control for the forward motion can be designed as

$$F = -\text{sign}(x_5) |x_5|^{1/3} - \text{sign} \left[ \left( x_2 + \frac{3}{5} x_5^{5/3} \right) \right] \left| \left( x_2 + \frac{3}{5} x_5^{5/3} \right) \right|^{1/5}.$$

This force control drives the state  $x_2 = 0$  in some finite time  $T_3 > 0$ . The closed-loop system can be written as:

$$\begin{aligned}
 \dot{x}_1 &= x_4 \\
 \dot{x}_2 &= x_5 - x_4 x_3 \\
 \dot{x}_3 &= x_4 x_2 \\
 \dot{x}_4 &= -k_1 K_1 \text{sign}(x_4) |x_4|^{1/3} \\
 \dot{x}_5 &= k_2 \left[ -\text{sign}(x_5) |x_5|^{1/3} - \text{sign} \left[ \left( x_2 + \frac{3}{5} x_5^{5/3} \right) \right] \left| \left( x_2 + \frac{3}{5} x_5^{5/3} \right) \right|^{1/5} \right]
 \end{aligned} \tag{6}$$

and for  $t \geq T_3 + T_2 + T_1$ , the closed-loop trajectories reach the set  $\mathcal{D}_4 = \{x : x_2 = x_3 = x_4 = x_5 = 0\}$ .

### 3.4. State 4: rotate to the desired orientation

At the end of the third stage, the robot is at the desired position but not with the desired orientation. The desired orientation is performed using the torque control and this can be considered as the regulation of the state  $x_1 = 0$  in finite time. Consider the dynamics of  $x_1$

$$\begin{aligned}
 \dot{x}_1 &= x_4, \\
 \dot{x}_4 &= k_1 \tau,
 \end{aligned}$$

with the following control law

$$\tau = -\text{sign}(x_4) |x_4|^{1/3} - \text{sign} \left[ \left( \sin(x_1 + \frac{3}{5} x_4^{5/3}) \right) \right] \left| \left( \sin(x_1 + \frac{3}{5} x_4^{5/3}) \right) \right|^{1/5}.$$

The closed-loop system is given by

$$\begin{aligned}
 \dot{x}_1 &= x_4 \\
 \dot{x}_2 &= x_5 - x_4 x_3 \\
 \dot{x}_3 &= x_4 x_2 \\
 \dot{x}_4 &= k_1 \left[ -\text{sign}(x_4) |x_4|^{1/3} - \text{sign} \left[ \left( \sin(x_1 + \frac{3}{5} x_4^{5/3}) \right) \right] \left| \left( \sin(x_1 + \frac{3}{5} x_4^{5/3}) \right) \right|^{1/5} \right] \\
 \dot{x}_5 &= -k_2 K_2 \text{sign}(x_5) |x_5|^{1/3}
 \end{aligned} \tag{7}$$

and the closed-loop trajectories reach the origin in some finite time  $T_4 > 0$ . At the end of fourth stage, the system is steered to the origin. Each control action can be summarized as a state-dependent switching control as follows.

With the following definitions  $u \triangleq [\tau F]^T$ ,  $\mathcal{P} \triangleq \{1, 2, 3, 4\}$ , the closed-loop switched system can be expressed as  $\dot{x} = f(x, u_\sigma)$  where the switching signal  $\sigma : \mathcal{M} \rightarrow \mathcal{P}$ , takes on the values based on the state-dependent switching criteria  $\sigma = i$  if  $x \in \mathcal{D}_i$ . The control strategy can now be summarized as

$$\begin{aligned}
 u_1 &= \begin{bmatrix} -K_1 \text{sign}(x_4) |x_4|^{1/3} \\ -K_2 \text{sign}(x_5) |x_5|^{1/3} \end{bmatrix} \\
 u_2 &= \begin{bmatrix} -\text{sign}(x_4) |x_4|^{1/3} - \text{sign} \left[ \left( \sin(x_{1t} + \frac{3}{5} x_4^{5/3}) \right) \right] \left| \left( \sin(x_{1t} + \frac{3}{5} x_4^{5/3}) \right) \right|^{1/5} \\ -K_2 \text{sign}(x_5) |x_5|^{1/3} \end{bmatrix} \\
 u_3 &= \begin{bmatrix} -K_1 \text{sign}(x_4) |x_4|^{1/3} \\ -\text{sign}(x_5) |x_5|^{1/3} - \text{sign} \left[ \left( x_2 + \frac{3}{5} x_5^{5/3} \right) \right] \left| \left( x_2 + \frac{3}{5} x_5^{5/3} \right) \right|^{1/5} \end{bmatrix} \\
 u_4 &= \begin{bmatrix} -\text{sign}(x_4) |x_4|^{1/3} - \text{sign} \left[ \left( \sin(x_1 - \frac{3}{5} x_4^{5/3}) \right) \right] \left| \left( \sin(x_1 + \frac{3}{5} x_4^{5/3}) \right) \right|^{1/5} \\ -K_2 \text{sign}(x_5) |x_5|^{1/3} \end{bmatrix}.
 \end{aligned}$$

Using the notion of flow, the control switching can be expressed as:

$$\phi_{f+g_1\tau}^{T_4} \circ \phi_{f+g_2F}^{T_3} \circ \phi_{f+g_1\tau}^{T_2} \circ \phi_{f+g_1\tau+g_2F}^{T_1}(x) = 0, x \in \mathcal{M}, \tag{8}$$

where  $\tau, F$  take on appropriate expressions based on the switching manifolds  $\mathcal{D}_i, i = 1, \dots, 4$ .

**Remark 1.** The switching function  $\sigma$  maps elements of  $\mathcal{M}$  to the set  $\mathcal{P}$ . Further, if the flows in (8) can be analytically found, then the state-dependent switching is equivalent to a time-dependent switching for a suitably defined time-dependent switching signal  $\tilde{\sigma} : [0, \infty) \rightarrow \mathcal{P}$ .

### 4. Stability and robustness analysis

In this section we present the stability analysis of the proposed switched control algorithms. Consider the Lyapunov candidate function

$$V(x) = \frac{1}{2}(x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2)$$

and further define

$$V_i(x) = \{V(x) : x \in \mathcal{D}_i\}.$$

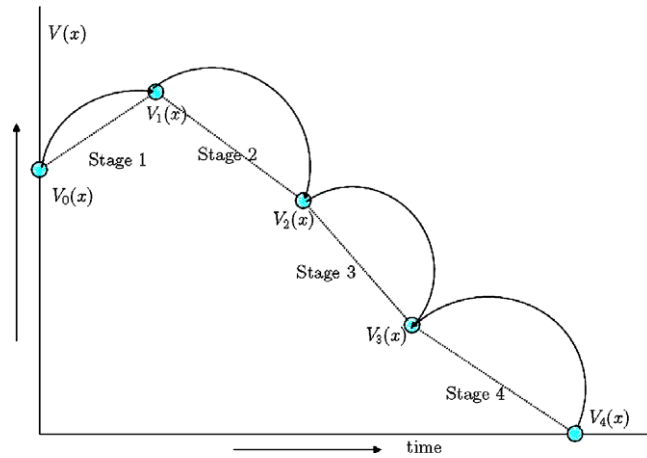


Fig. 2. Lyapunov function.

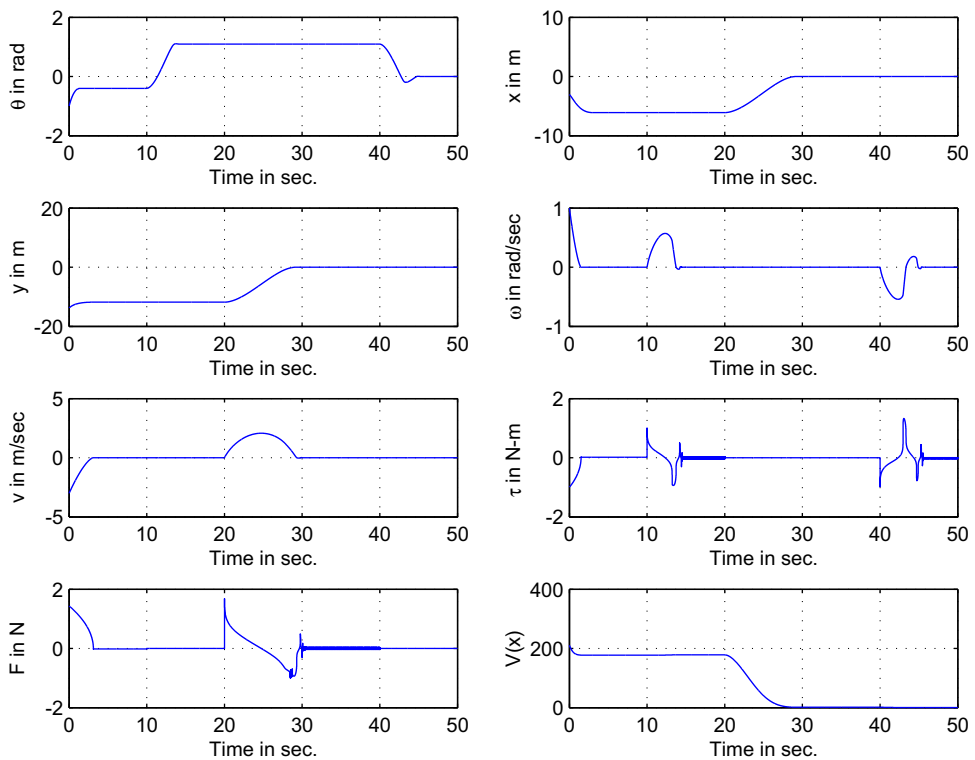


Fig. 3. States and control inputs – thin switching surface.

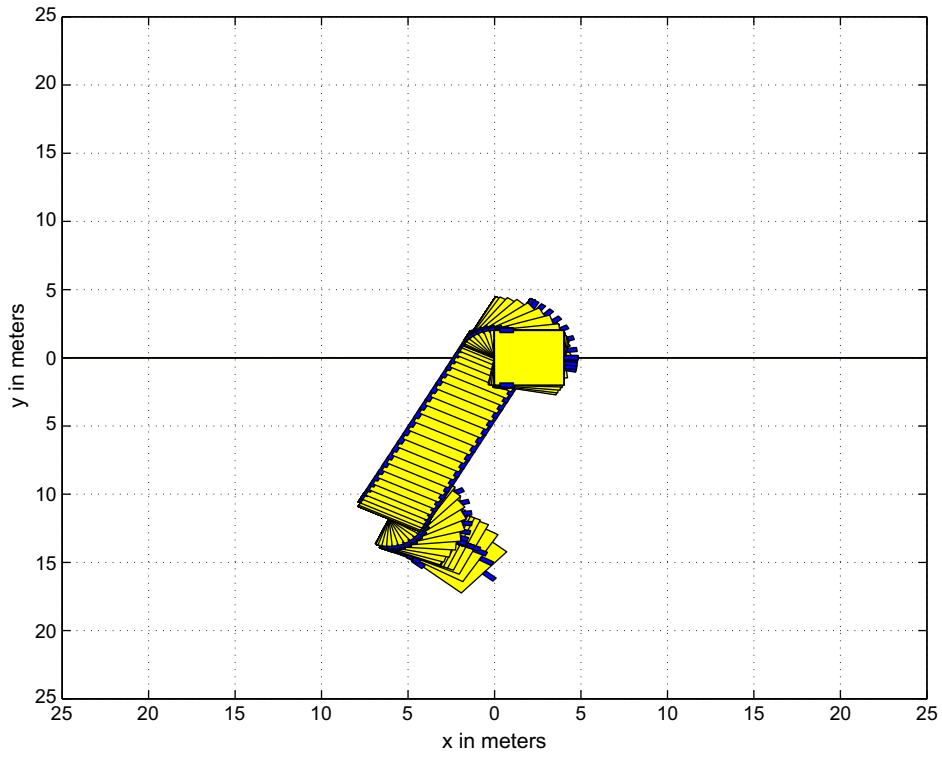


Fig. 4. Path traced by the mobile robot.

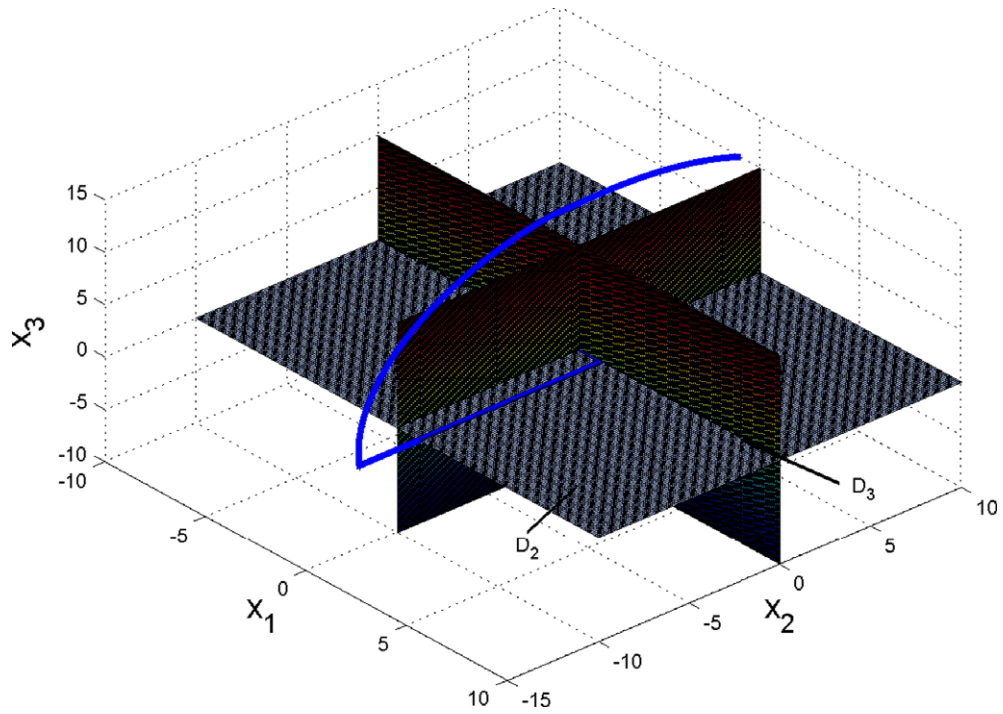


Fig. 5. Thin switching surfaces.

It is easy to note that the following holds

$$V_4(x) < V_3(x) < V_2(x) < V_1(x)$$

$$V_4(x) < V_3(x) < V_2(x) < V_0(x)$$

and further the time histories of  $V_i$ 's can be depicted as shown as in Fig. 2 from which it is easy to conclude that for every  $V_0(x)$ , there exists constants  $K_{x_0}, k_{x_0} > 0$  depending on the initial condition  $x(0) = x_0$  such that

$$V(x) \leq K_{x_0} V(x(0)) e^{-k_{x_0} t}.$$

Since the constants  $K_{x_0}, k_{x_0}$  depend on the initial condition, we can infer, at the most, the convergence of the closed-loop trajectories to the equilibrium, and not in the sense of conventional stability. We next present some robustness analysis to validate the proposed switched control algorithm. It has been shown that the switched control methods are robust with parametric uncertainty in [11].

In the proposed approach, we have defined thin switching surfaces in which the controller switches from one to another. The thin switching surface is the conventional switching surface used in the sliding-mode terminology. It is not practically feasible to switch on the thin surface. Moreover, with various uncertainties it is not feasible to reach such thin surfaces. To mitigate this problem and from the point of view of the controller implementation, we redefine the switching surface, termed *relaxed switching surface* as follows:

$$\mathcal{D}_{1r} \triangleq \mathcal{M},$$

$$\mathcal{D}_{2r} \triangleq \{x : |x_4|, |x_5| \leq \epsilon\},$$

$$\mathcal{D}_{3r} \triangleq \{x : |x_3|, |x_4|, |x_5| \leq \epsilon\},$$

$$\mathcal{D}_{4r} \triangleq \{x : |x_2|, |x_3|, |x_4|, |x_5| \leq \epsilon\}, \quad \epsilon > 0.$$

The relaxed switching surfaces can be viewed as strips around the thin sets  $\mathcal{D}_i, i = 1, \dots, 4$ . The relaxed switching surface retains the positive invariance property even in the presence of uncertainties and small disturbances unlike the thin sets. However, the use of relaxed switching surface guarantees only practical stability.

The switching signal is now redefined as  $\sigma = i$  if  $x \in \mathcal{D}_{ir}$ . We next present simulation results of the closed-loop system based on the thin as well as the relaxed switching surfaces.

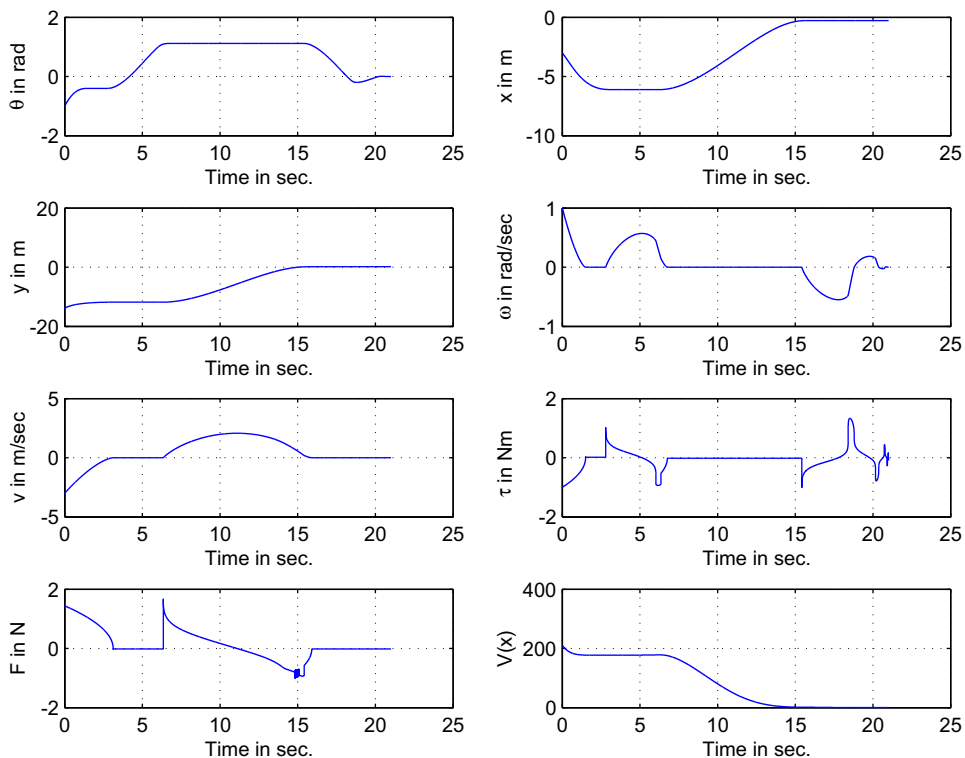


Fig. 6. States and control inputs – relaxed switching surface.



## 5. Simulation results

We simulate the proposed controller for the initial condition  $\theta(0) = -1$  rad,  $x(0) = -3$  m,  $y(0) = -13.8$  m,  $\omega(0) = 1$  rad/s,  $v(0) = -3$  m/s. The time response using the switching regions  $\mathcal{D}_i$  is shown in Fig. 3. The robot path is shown in Fig. 4 and the switching surface is depicted in Fig. 5. The time-response of the control strategy with the relaxed switching surface, using  $\epsilon = 0.01$  is shown in Fig. 6.

## 6. Conclusions

We have presented a switched control strategy to steer a mobile robot from one point to another. The physical task of stop, home, move and re-orient is mathematically related in terms of different operating regions, thereby rendering the control strategy amiable for state-dependent switched control. To obviate the drawback of thin switching surfaces, we have introduced relaxed switching surface and demonstrated the effectiveness of the controller using simulations.

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